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Social Research Methods Leeds
Interdisciplinary Centre

What Is... Measurement Error?

Jose Pina-Sánchez

Introduction

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  - Discrepancies between the ‘true’ and the ‘observed’ value
  - Flaws in the foundations of quantitative research that can bias our findings severely

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  - What forms does it take?
  - How can it bias our findings?
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- Side note: stop me whenever you have any questions, and be ready as I will be asking questions too

Where Can We Expect It?

- Some examples:
  - E.g.1 survey data (*what are your monthly earnings?*)
  - E.g.2 administrative data (*using police data to measure violence*)
  - E.g.3 content analysis (*ethnicity derived from names*)

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  - Question: What other cases have you encountered?
- These errors are not always equivalent

Types of Measurement Error

- Random errors (the classical measurement error model)
  - $\overbrace{X^*}^{observed} = \overbrace{X}^{true\ value} + \overbrace{U}^{noise}$
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- Systematic errors
  - $X^* = X + U$; but $E(U) \neq 0$

Multiplicative Errors

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  - I.e., as $X^* = X \cdot U$, rather than $X^* = X + U$
- And if the data is categorical, then we have misclassification
  - For a binary variable

$$\begin{cases} P(X^* = 1|X = 1) = \theta_{1|1}; & \text{Sensitivity} \\ P(X^* = 0|X = 0) = \theta_{0|0}; & \text{Specificity} \end{cases}$$

$$\begin{cases} P(X^* = 1|X = 0) = \theta_{1|0}; & \text{Probability false positive} \\ P(X^* = 0|X = 1) = \theta_{0|1}; & \text{Probability false negative} \end{cases}$$

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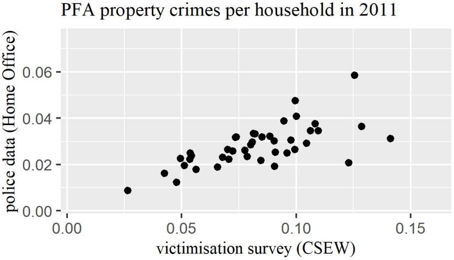
  - Examples from your own research
  - Using police data to measure the prevalence of hate crime

Multiple Error Mechanisms

- Often variables are affected by multiple measurement error mechanisms
- This is how we define measurement error in police data
  - *systematic*, since not all crime is reported to the police
  - *random*, subject to variability across areas, as a result of the different recording practices across police forces
  - *multiplicative*, errors seem proportional to the true extent of crime in the area

Multiplicative Errors: Crime Rates



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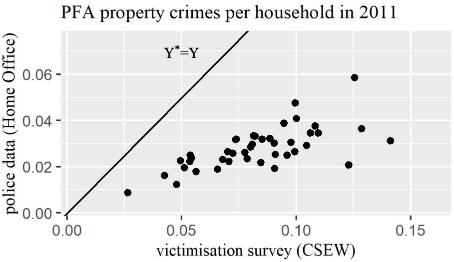

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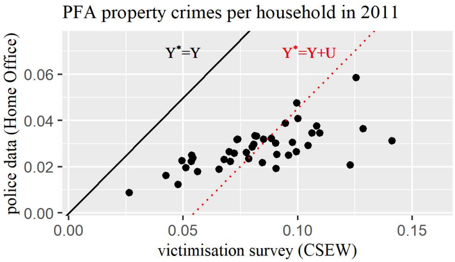

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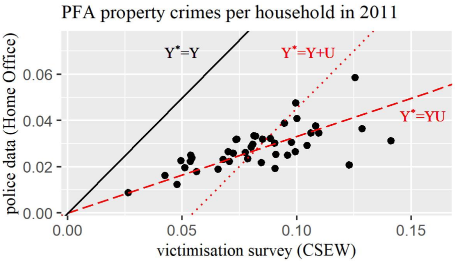

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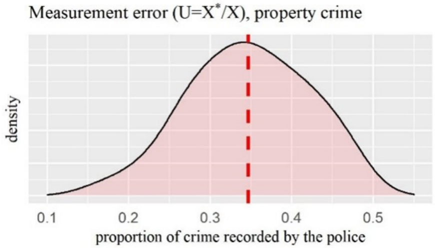

The Impact of Measurement Error

- We can see how different forms of measurement error affect univariate stats
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- But how does measurement error affect estimates from multivariate (regression) models?

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- We can see how different forms of measurement error affect univariate stats
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- But how does measurement error affect estimates from multivariate (regression) models?
- Assuming only one variable is prone to measurement error, its impact will depend on:
  1. the outcome model (whether linear, Poisson, etc.)
  2. the measurement error model (additive, random, etc.)
  3. where is the affected variable introduced in the model (as a response or an explanatory variable)

Impact of Measurement Error

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  - $Y = \alpha + \beta X + \epsilon$

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- Question: Will β be biased in any of those scenarios?

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    $Y = \alpha + \beta X + (\epsilon - U)$
  - The measurement error is absorbed by the model's error term, affecting precision, but leaving regression coefficients unbiased
  - We can see this effect using simulated data

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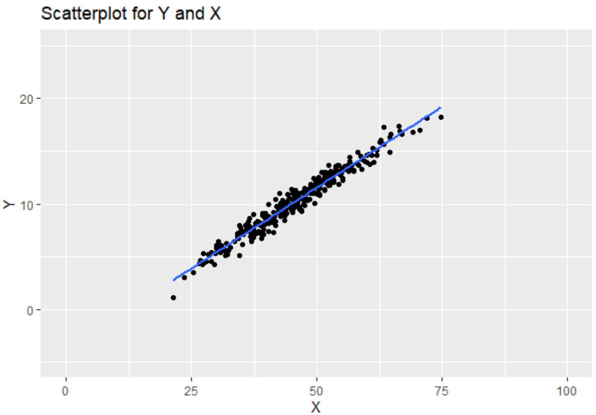

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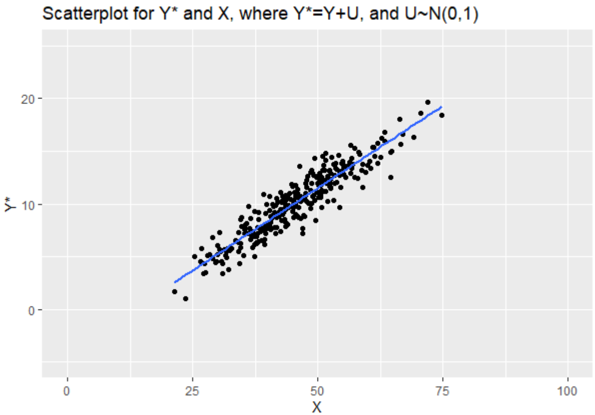

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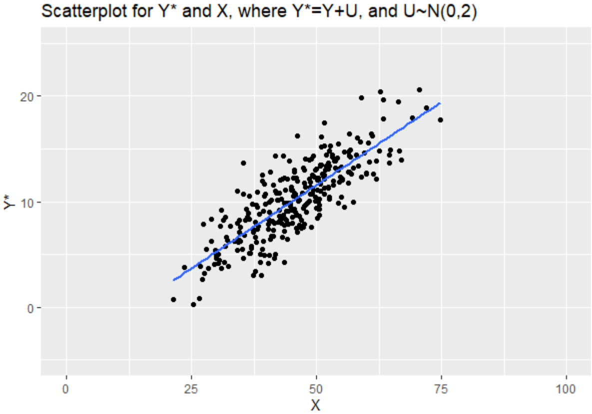

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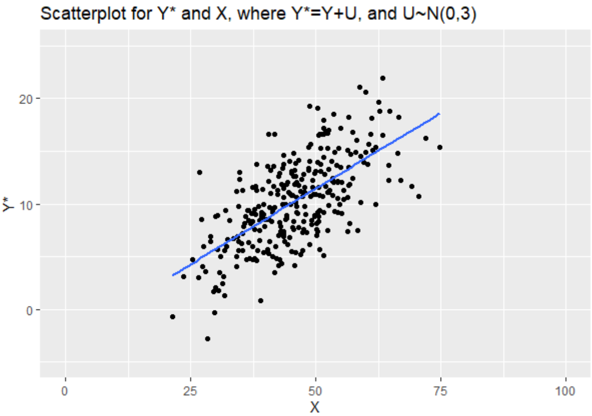

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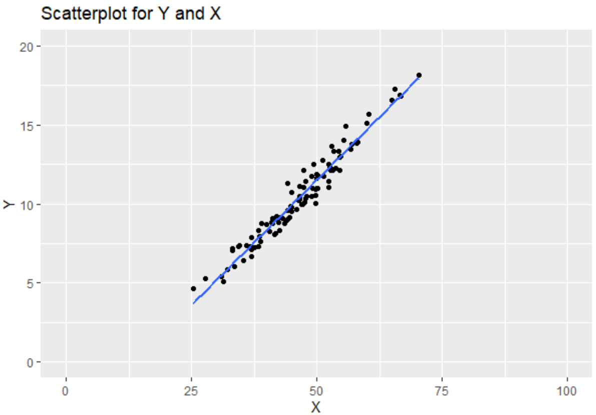

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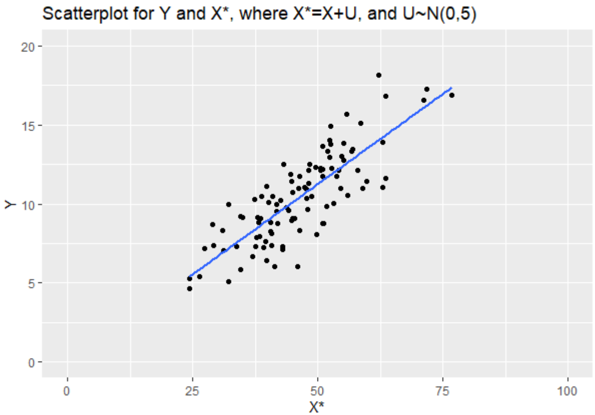

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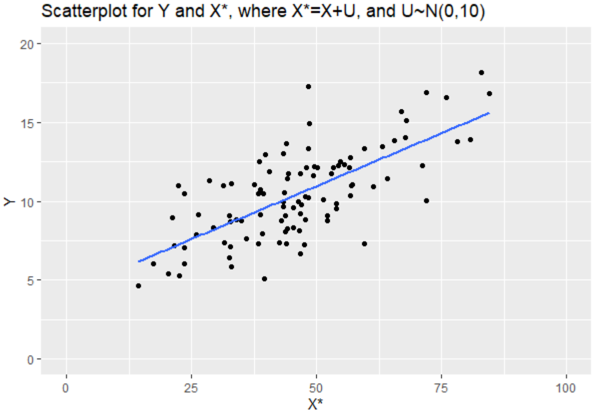

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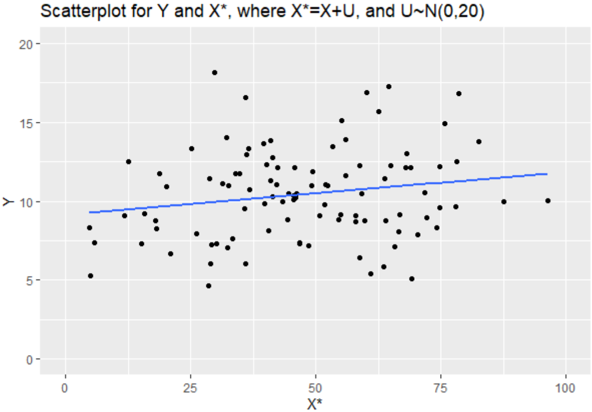

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    $Y \cdot U = \alpha + \beta X + \epsilon$

    $Y = \dfrac{\alpha + \beta X + \epsilon}{U}$
  - All regression coefficients are biased

Impact of Measurement Error

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- Depending on the type of errors, we see vastly different impacts
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- And we have only considered relatively simple scenarios
- In the words of Nugent et al. (2000: 60):
  - *"Measurement error is, to borrow a metaphor, a gremlin hiding in the details of our research that can contaminate the entire set of estimated regression parameters"*

Adjustment Methods

- We should always aim to improve data collection processes
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- We should always aim to improve data collection processes
- When that is not possible/sufficient we should adjust for the impact of measurement error
- We have seen how in some simple settings we can anticipate - and therefore adjust - that impact
- When we can't trace out the impact of measurement error algebraically we need to use other methods

Adjustment Methods

- Most adjustment methods require additional forms of data
  - Multiple reflective indicators (latent variable models)
  - Instrumental variables (two stage processes)
  - A validation subsample (multiple imputation)
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  - A validation subsample (multiple imputation)
  - Repeated observations (regression calibration)
- Others can be used when all you have is an educated guess (sensitivity analysis)
  - Bayesian adjustments (Gustaffson, 2003)
  - Multiple overimputation (Blackwell et al., 2017)
  - SIMEX (Cook & Stefanski, 1994)
  - Simulations (the *RCME* package Pina-Sánchez et al., 2022)

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